

AUTOMATED TWO- DIMENSIONAL FUZZY C-MEANS WITH K-MEANS CLUSTERING FOR IMAGE SEGMENTATION

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ABSTRACT

Segmentation is a technique that divides an image into many regions that have strong relations with objects to demonstrate the real information collected from the real world. We presented hybrid method of A2DFCM and K-means clustering algorithm for image segmentation. This paper introduces the Automated Two-Dimensional Fuzzy C-mean with K-means (A2DFCM-KM) algorithm. It is a novel unsupervised clustering technique. The proposed technique differs from the conventional clustering technique because user needs not to be determining the number of clusters. A2DFCM-KM relates to local and spatial information of the data into the clustering analysis. By this proposed method we minimized CPU time to about one sixth compare to other existing methods without affecting the overall quality of terminal clusters produced. During iteration we will reduce an objective function that shows the distance between data points and cluster center weights using membership value. Simulation results show that the proposed method outperforms the existing A2DKM method qualitatively as well as quantitatively as it is less sensitive to noise and also eliminates noisy spots. The results also shows that the proposed method reduces false blobs and also more homogeneous regions are obtained by it.

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1. INTRODUCTION

Image segmentation is an ongoing major research topic for many image processing researchers. The rapid development in image processing technology is possible with image segmentation. Image segmentation plays an important role in image analysis and computer vision, because there is no technique that can handle different type of image segmentation. The main goal of image segmentation is to divide an image into set of disjoint region, which can be considered homogenous attributes such as color, motion, texture, tone, etc [23, 24]. Many different segmentation techniques have been developed by many researchers [1-6]. The image segmentation technique can be divided into four categories, edge detection, clustering, region extraction and thresholding. In this paper, a clustering based method for image segmentation is considered.

Many clustering technique have been used, such as the crisp clustering scheme and the fuzzy clustering scheme, each of which has its own special characteristics [7]. The conventional crisp clustering method restricts each point of the data set to exclusively just one cluster. There are many real conditions for images, such as issues of noise, overlapping intensities, limited spatial resolution, poor contrast and intensity in homogeneity variation makes crisp segmentation a difficult task. The fuzzy set theory [8], which involves the idea of partial membership described by a membership function, introduces fuzzy clustering as a soft segmentation method that has been widely studied and successfully applied to image segmentation [10 – 12]. Among the fuzzy clustering methods, fuzzy C-means (FCM) algorithm [9] is the most popular method used in image segmentation because it has robust characteristics for ambiguity and can retain more information than hard segmentation methods [10, 11]. Although the conventional FCM algorithm works well on most noise-free images, it has a serious limitation: it does not incorporate any information about spatial context, which reason it to be sensitive to noise and imaging artifacts. To compensate for this drawback of FCM, the obvious way is to smooth the image before segmentation. Though, the conventional smoothing filters can result in loss of important image details, especially boundaries or edges of image. The KM clustering algorithm clusters data into a fixed number of clusters using the Euclidean distance based on the pixel's intensity value.

2. RELATED WORK

In our proposed work we have presented a hybrid method of Automated Two- Dimensional Fuzzy C-means with K-means clustering for Image Segmentation .There are several research that has been carried out in the this field as follows. Yusoff, Nor and Ashidi Mat Isa [26] introduces the Automated Two-Dimensional K-Means (A2DKM) algorithm, a novel unsupervised clustering technique. The proposed technique differs from the conventional clustering techniques because it eliminates the need for users to determine the number of clusters. In addition, A2DKM incorporates local and spatial information of the data into the clustering analysis. A2DKM is qualitatively and quantitatively compared with the conventional clustering algorithms, namely, the K-Means (KM), Fuzzy C-Means (FCM), Moving K-Means (MKM), and Adaptive Fuzzy K-

Means (AFKM) algorithms. The A2DKM outperforms these algorithms by producing more homogeneous segmentation results. DespotoviIvana [16] presents a new FCM-based method for spatially coherent and noise-robust image segmentation. The contribution is twofold: 1) the spatial information of local image features is integrated into both the similarity measure and the membership function to compensate for the effect of noise; and 2) an anisotropic neighborhood, based on phase combination features, is introduced to allow more accurate segmentation without image smoothing. The segmentation results, for both synthetic and real images, show that our method efficiently preserves the homogeneity of the regions and is more robust to noise than related FCM-based methods. Sudhavani and Sathyaparsad [13] presented modified fuzzy C-Means clustering algorithm for lip image segmentation. Kannan et al. [14] have developed an efficient fuzzy segmentation algorithm for breast magnetic resonance imaging data. They have acquired an objective function of FCM called Kernel Induced FCM which is based on hyper tangent function which is in turn based on two functions namely kernel, hyper tangent and Lagrangian multipliers. Huynh Van Lung and Jong-Myon Kim [15] have developed generalized Spatial Fuzzy C-Means Clustering algorithm (GSFCM) for brain MRI segmentation. GSFCM utilizes given pixel attributes and spatial local information weighted equally to neighbors based on their distance attributes. Results have shown that GSFCM outperforms conventional FCM. Huiyu Zhou et al. [16] have presented a mean shift based FCM for the extraction of skin lesion. They have proposed a mean shift based fuzzy c-means object function that is a mean field term is incorporated in the standard FCM objective function. Experimental results have shown that their algorithm is capable of extracting skin lesion borders proficiently. Ruoyu Du and Hyo Jong Lee [17] have proposed an enhanced segmentation technique that applies sigma filter to vary neighboring pixels of target. Visual and quantitative assessment has exposed that the proposed method works better than the original FCM.

Clustering is traditionally viewed as an unsupervised method for data analysis. However, in some cases information about the problem domain is available in addition to the data instances themselves. Kiri Wagstaff et al. [23] proposed a work in which they demonstrate how the popular k-means clustering algorithm can be profitably modified to make use of this information. In experiments with artificial constraints on six data sets, they observed improvements in clustering accuracy. They also applied this method to the real-world problem of automatically detecting road lanes from GPS data and observed dramatic increases in performance.

Practical approaches to clustering use an iterative procedure (e.g. K-Means, EM) which converges to one of numerous local minima. It is known that these iterative techniques are especially sensitive to initial starting conditions. P. S. Bradley et al. [24] presented a procedure for computing a refined starting condition from a given initial one that is based on an efficient technique for estimating the modes of a distribution. The refined initial starting condition allows the iterative algorithm to converge to a “better” local minimum. The procedure is applicable to a wide class of clustering algorithms for both discrete and continuous data. They demonstrated the application of this method to the popular K-Refinement run time is considerably lower than the

time required to cluster the full database. The method is scalable and can be coupled with a scalable clustering algorithm to address the large-scale clustering problems in data mining.

AristidisLikas et al.[25] presented the global k-means algorithm which is an incremental approach to clustering that dynamically adds one cluster center at a time through a deterministic global search procedure consisting of N(with N being the size of the data set) executions of the k-means algorithm from suitable initial positions. Also they have proposed modifications of the method to reduce the computational load without significantly affecting solution quality. Their clustering methods are tested on well-known data sets and they compare favorably to the k-means algorithm with random restarts.

(i) Color Image Segmentation Using Fuzzy C-means Clustering

In Fuzzy clustering (also referred to as soft clustering), data elements can belong to more than one cluster, and associated with each element is a set of membership levels. These indicate the strength of the association between that data element and a particular cluster. Fuzzy clustering is a process of assigning these membership levels, and then using them to assign data elements to one or more clusters.

FCM is the most widely used color image segmentation technique. Some of the limitations associated with FCM [20]:

- In terms of performance FCM depends on the initial cluster center.
- The cluster number must be fixed before clustering.
- Computational complexity is more.
- Spatial information is not considered in conventional FCM algorithm.

(ii) Color Image Segmentation Using K -means Clustering

The K-means algorithm is an unsupervised clustering data mining/machine learning algorithm that is used to divide an image into k clusters. The main purpose of clustering techniques is to partition a set of entities into different groups, called clusters. These groups can be consistent for terms of similarity of its members. The k-means algorithm is one of the simplest clustering techniques and it is commonly used in medical imaging, biometrics and related fields.

Steps in the algorithm are as follows:

Step1. Choose the number k of clusters, either randomly or based on some heuristic.

Step2. Generate k clusters and determine the cluster center.

Step3. Assign each pixel in the image to the clusters that minimize the distance between the pixel and the cluster center (Distance is the squared or absolute difference between a pixel and a cluster center).

Step4. Re-compute cluster center by averaging all of the pixels in the cluster.

Step5. Repeat steps 2 and 3 until convergence is attained(for example cluster center remains unchanged).

Disadvantages of K-means algorithm

- Difficult to predict k with fixed number of clusters.
- Does not work well with non-globular cluster.

3. PROPOSED METHODOLOGY

In A2DFCM-KM clustering with spatial information is proposed for image segmentation. In the image, the nearest pixels are closely associated to each other and there is superior probability that they fit in to the same group or cluster. In conventional FCM this spatial Information is ignored which is an important parameter for clustering. In proposed scheme, we worked on spatial information for membership calculation and it provides better clustering centers. By using KM we get quantitative parameters.

Proposed Technique:

In segmenting an 8-bit gray scale digital image with 256 gray levels within the interval [0,255] using the proposed

A2DFCM-KM algorithm, let $p(x,y)$ be the pixel at location (x,y) with an intensity p (where $x = 1, 2, 3, \dots, R$ and $y = 1, 2, 3, \dots, S$, with R and S as the number of columns and rows of the image, respectively). The initial values for all clusters are predetermined, and the number of clusters is initially set to two.

Generally, In conventional FCM clustering algorithm, the probability is dependent solely on the distance between the pixel and each individual cluster center in the feature domain. The membership functions and cluster centers are updated by the following:

$$u_{ij} = \frac{1}{\sum_{k=1}^c \left(\frac{\|x_j - v_i\|}{\|x_j - v_k\|} \right)^{2/(m-1)}} \quad (1)$$

is defined as membership of pixel x_j to the cluster j .

$$v_i = \frac{\sum_{j=1}^N u_{ij}^m x_j}{\sum_{j=1}^N u_{ij}^m}$$

where $u_{ij} \in [0, 1]$. m is a constant that defines the fuzziness, ϵ is a constant and its value is manually determined in

the range of $[0.01, 0.0001]$. Starting with an initial guess for each cluster center, the FCM converges to a solution for v_i representing the local minimum or a saddle point of the cost function. Convergence can be detected by comparing the changes in the membership function or the cluster center at two successive iteration steps.

The proposed A2DFCM-KM clustering spatial information is fused in the membership function to obtain better segmentation results and is defined as follows:

$$ij = \frac{u_{ij}^a h_{ij}^b}{\sum_{k=1}^c u_{kj}^a h_{kj}^b} \quad (2)$$

where a and b are constants to supervise the relative importance of both spatial and membership functions. The spatial function is given as:

$$h_{ij} = \sum_{k \in NB(x_j)} u_{ik}$$

where $NB(x_j)$ is a square window centered on pixel (x_j) in the spatial domain.

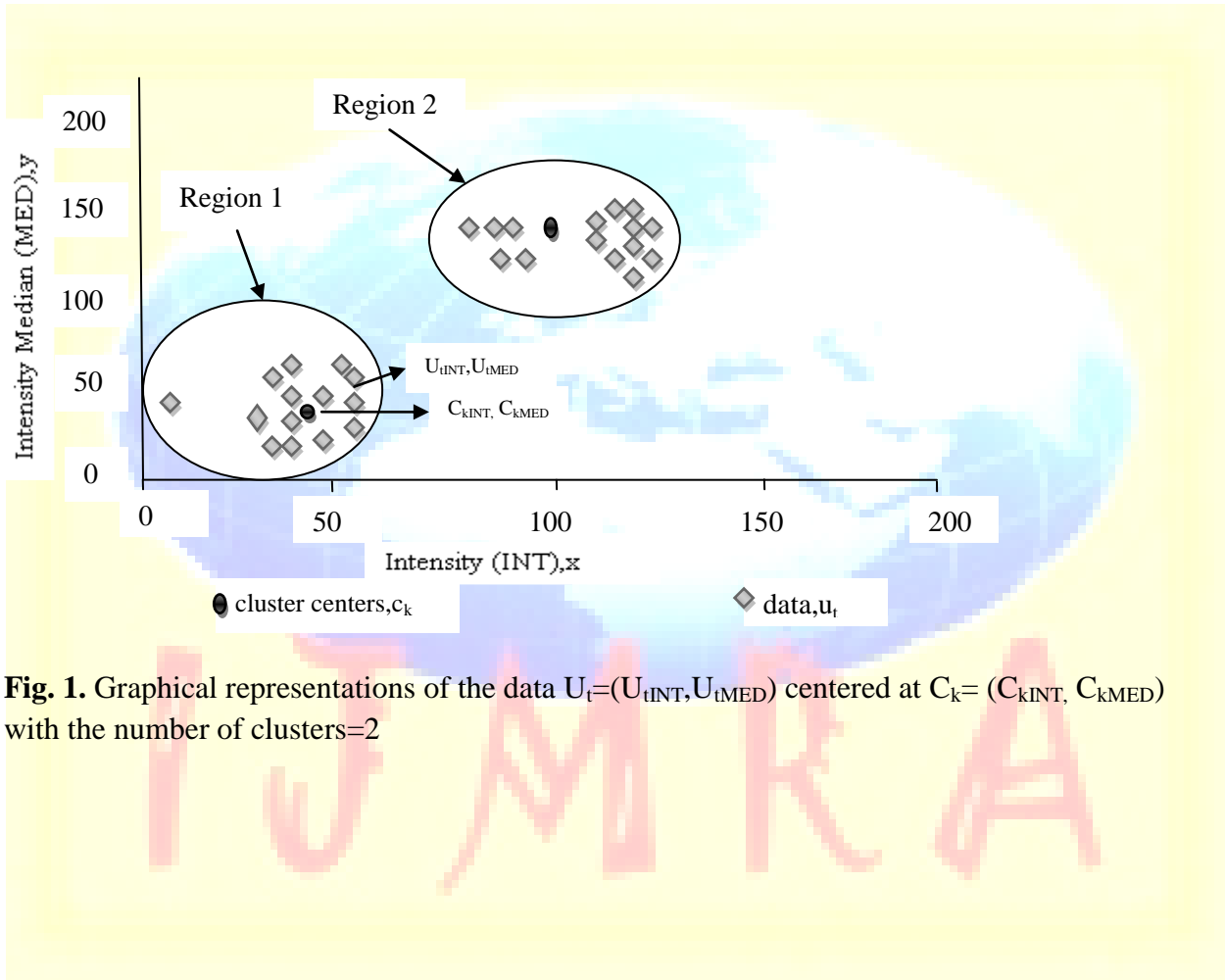


Fig. 1. Graphical representations of the data $U_t=(U_{iINT},U_{iMED})$ centered at $C_k=(C_{kINT}, C_{kMED})$ with the number of clusters=2

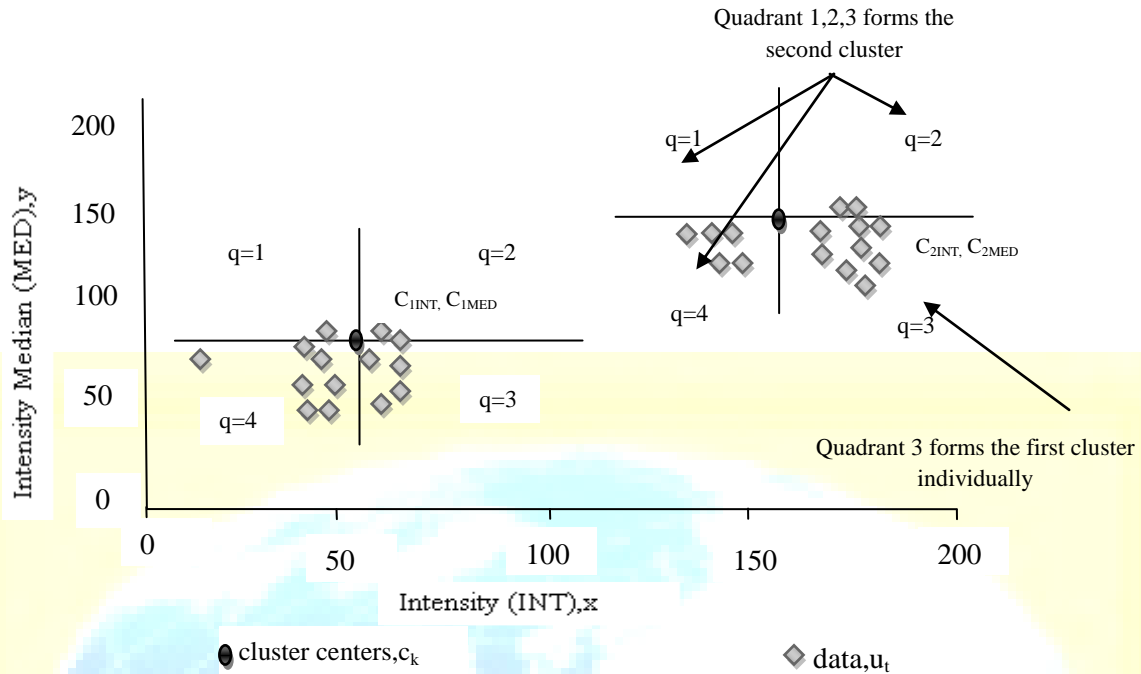


Fig. 2. Visualization of two quadrant axes built for two initial clusters that are centered at (C_{1INT}, C_{1MED}) and (C_{2INT}, C_{2MED}) .

The graphical two-dimensional annotation of data with two separated regions, along with the aforementioned parameters, is illustrated in Fig 1. U_{tINT} is the intensity (INT) dimension of t^{th} data, U_{tMED} is the intensity median (MED) dimension of t^{th} data, and n_k is the number of pixels assigned to k^{th} center.

In the proposed A2DFCM-KM algorithm, break the running clustering process .After breaking the process we applied A2DFCM-KM algorithm and done the process of reclustering .Then each region of the k^{th} cluster is split into four quadrants ($q = 1, 2, 3, 4$) by setting the origin at (C_{kINT}, C_{kMED}) , as shown in Fig. 2. The calculation of membership function (U_{kq}) using total Euclidean distances of all members in each quadrant is obtained by using:

$$U_{kq} = \sum_{k=1}^c \sum_{q=1}^n U_{kq}^m d_{kq}^2 \tag{3}$$

Where,

$$d_{kq} = \sum_{i \in C_{kq}} \| U_i - C_{kq} \| \quad \text{for } q = 1, 2, 3, 4 \tag{4}$$

(4)

where C_{kq} is the q^{th} quadrant of the k^{th} cluster, and U_i is the i^{th} member of C_{kq} . The quadrant with the largest total Euclidean distance is then determined and denoted as d_{kqMAX} . The total Euclidean distances of all four quadrants are compared with the d_{kqMAX} to fulfill the following condition:

$$d_{kqMAX} < 0.0001(\sum_{q=1}^4 d_{kq}) \tag{5}$$

(5)

- If (5) is not fulfilled, then the k^{th} center splits and automatically converted into two new clusters.
- The first cluster consists of all the members of the quadrant having more no of pixels, while the other three quadrants having less no of pixels form the second cluster Fig. 2
- The processes of splitting the clusters into quadrants and forming new clusters are continues till the maximum difference between cluster centers at two successive iterations becomes less than a threshold= 0.02.to make (5) fulfill.
- After fulfilling of (5), no further formation of new clusters is essential.
- This process could automatically transform new clusters and determine the number of clusters
- The last number of clusters that was calculated is said to be the final (or optimum) cluster number.
- The process of clustering starts with two clusters as the initial value because a data used must be partitioned into at least two groups with different cluster centers for the clustering analysis to be meaningful.
- In this A2DFCM-KM algorithm, efficiency of pixels increases by which duration of execution time decreased

Steps involved in A2DFCM-KM clustering are as follows:

Step1. In first pass split the cluster to form two new clusters and than break clustering process do reclustering and than the membership function is computed.

Step2. In the second pass, the membership information of each pixel is delineated to the spatial domain, and the spatial function is determined from that.

Step3. During reclustering clusters automatically splits to form new clusters .The clusters are taken in the form of quadrants then automatically the quadrants with less pixels combine and form a new cluster and the quadrant with more pixels converted into another new cluster.

Step4. Next the A2DFCM-KM iterative procedure continues until some union is reached that is when the maximum difference between cluster centers at two successive iterations is less than a threshold = 0.02.

Step5. Now by K-mean method we find matrix of final cluster centers where each row provides the center coordinates and final fuzzy partition matrix.

Step6. Finally de-fuzzification method is used to allocate each pixel to a separate cluster for which the membership is maximal.

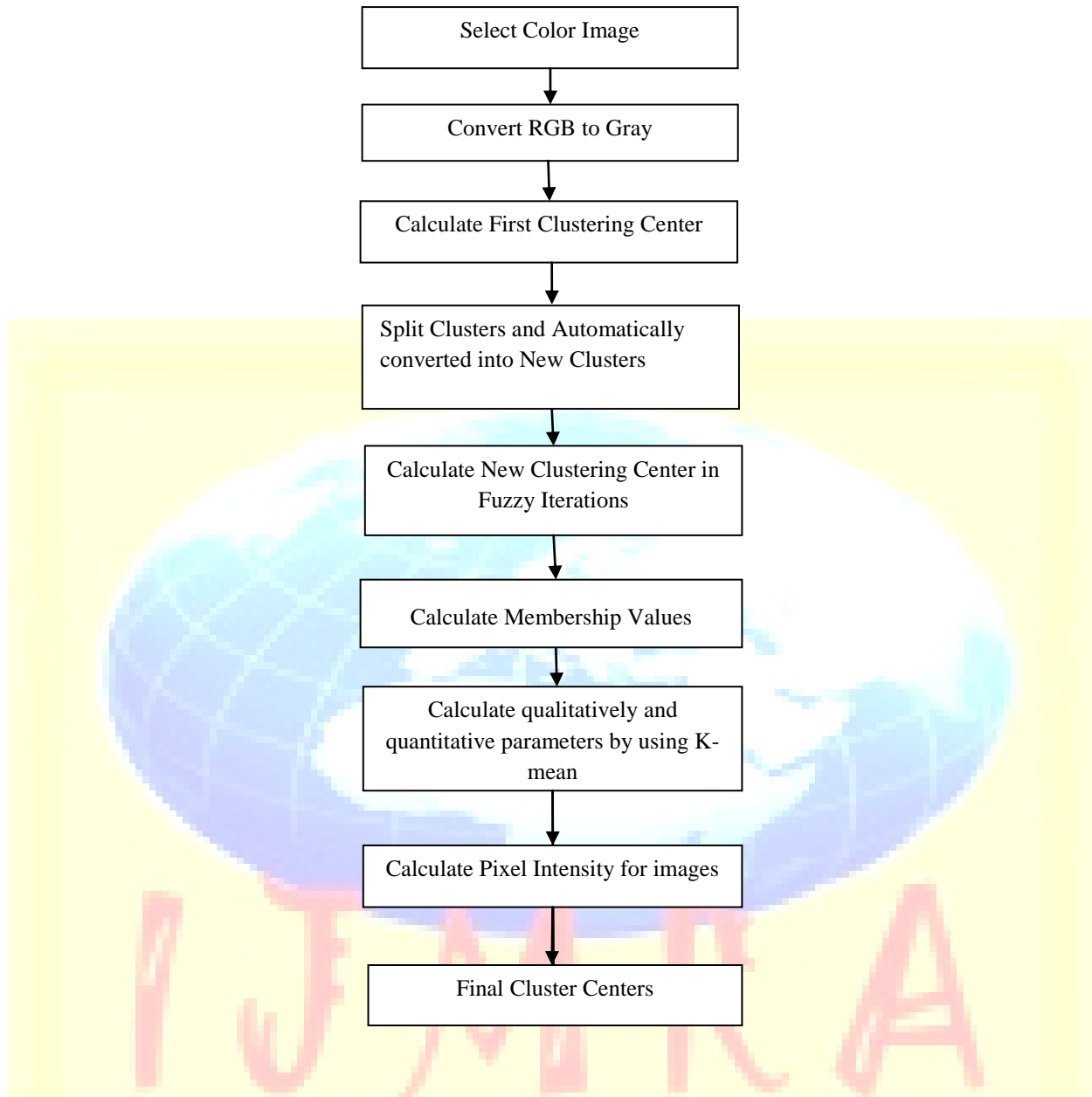


Fig.4 Flow Chart of Proposed System

The present work is focused on better enhancement to achieve improved segmentation. To retrieve important features to benefit the clustering process, first we select a color image (original image) and then this image from RGB into Gray. After this process we calculate the first center on the basis of clustering points. Split the clusters than splitted clusters automatically converted into new clusters. Calculate the new centers using A2DFCM-KM Clustering algorithm, and we calculate the membership value. In next step calculate the pixel intensity for images Fig.4. After completing, all this process of found final cluster centers. By using K-mean we get quantitative parameters, namely, $F(I)$, $F'(I)$, $Q(I)$.

The quantitative parameter is calculate using KM

$$F_{KM} = (\text{sqrt}(R) * E) / 1000 \times (NXM)$$

Where R is number of regions in an image, N X M is size of the image, F_{KM} is image parameter and E is Euclidean distance

$$E = \text{sqrt}(\text{sum}(IM - IMMM).^2)$$

Where IM is gray image and IMMM is segmented image

$$F'_{KM} = \text{Dist} / 10000 * (NXM)$$

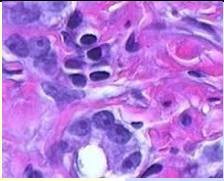
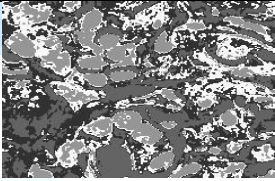
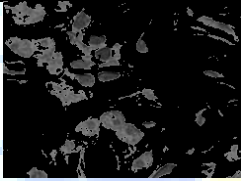
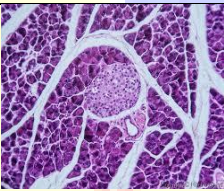
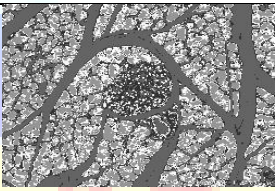
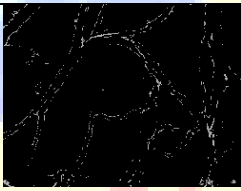
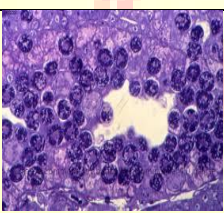
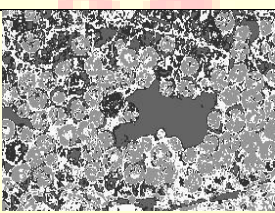

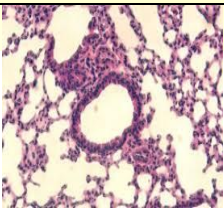
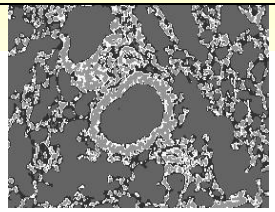
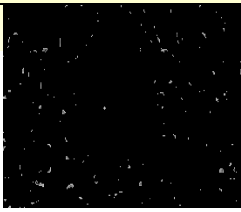
Where $\text{Dist} = e + e.^2 / \text{sqrt}(A) \times R$ and F'_{KM} is image parameter

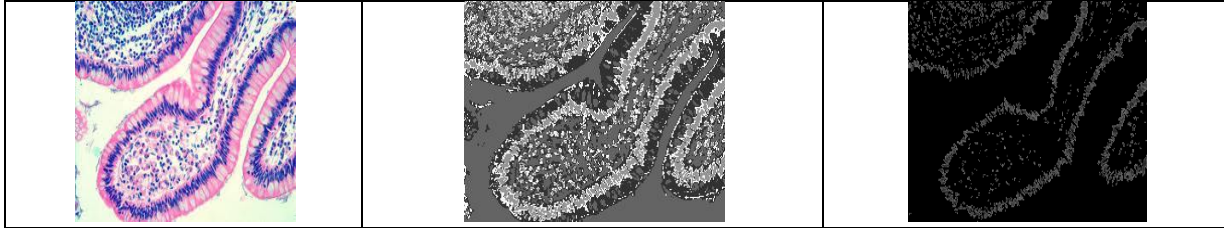
$$Q_{KM} = \text{sqrt}(R) * E / 10000 \times (NXM)$$

Where Q_{KM} is image parameter

4. Experimental Results.

Table 1

Original Image	Final Segmented Image A2DFCM-KM	Final Segmented Image A2DKM
		
		
		
		



Effects of A2DFCM-KM

The first part of the result is to prove that the proposed A2DFCM-KM clustering algorithm is able to give the optimum number of clusters for the clustering process. The proposed A2DFCM-KM is tested against original images, and is able to accurately estimate the number of clusters that is suitable for each image. The A2DFCM-KM clustering processes for 5 original images are summarized in Table fig 1, with the number of expected clusters and the resultant number of clusters from A2DFCM-KM. This clustering algorithm eliminates noisy spots, reduces false blobs, less sensitive to noise and more consistent regions are obtained.

5.RESULT ANALYSIS

The corresponding quantitative results for the different Hestain images are presented below each image in table 3, respectively. The best results are highlighted in bold. Therefore, from the detailed analysis of the five standard consumer images, the proposed A2DFCM-KM clustering algorithm is able to automatically determine the optimal number of clusters for segmentation. In addition, the proposed A2DFCM-KM clustering algorithm qualitatively performs better as compared with the A2DKM clustering algorithms by using the number of clusters which have been automatically obtained at the earlier stage of segmentation. F1, F2 and Q1 are parameters of level segmentation and T is for execution time. The proposed A2DFCM-KM clustering algorithm gives better F1 and F2 values than the A2DKM clustering algorithm for the hestain image, with a less than 2% difference in the results.

This result analysis states that the local and spatial distinctiveness that were engaged in the proposed A2DFCM-KM clustering algorithm are highly capable of reducing the segmentation of irrelevant regions, thereby ensuring more homogeneous segmented regions. The A2DFCM-KM clustering algorithm execute the process for determining the number of clusters as well as the clustering of pixels is a constant and cyclic process, till the optimum number of clusters is obtained. Thus, a longer processing time is inevitable because it has to include the time taken for the algorithm to obtain the number of clusters, as well as the time required for the actual clustering to occur.

In the result analysis, four parameters are working as the result analysis benchmarks, namely, $F(I)$, $F'(I)$, $Q(I)$, and the execution time t .

Table.2

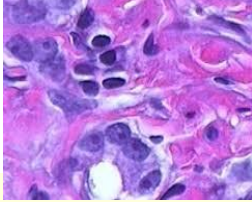
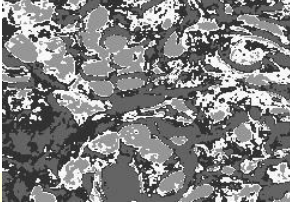
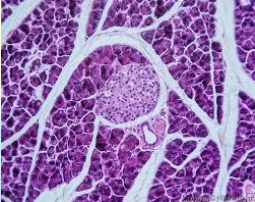
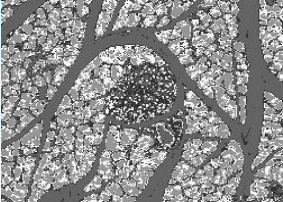
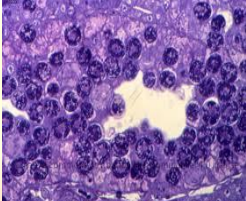
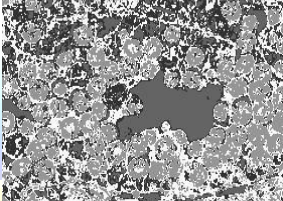
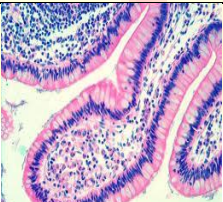
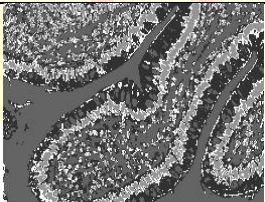
Original Image	Final Segmented ImageA2DFCM-KM
 <p>(a)</p>	 <p>No. Of Regions=210, $F(I) = 161.37$, $F'(I) = 99.62$, $Q(I) = 284.87$, $T(s) = 5.22$</p>
 <p>(b)</p>	 <p>No. Of Regions=224, $F(I) = 158.62$, $F'(I) = 98.37$, $Q(I) = 279.12$, $T(s) = 5.41$</p>
 <p>(c)</p>	 <p>No. Of Regions=324, $F(I) = 158.62$, $F'(I) = 98.37$, $Q(I) = 279.12$, $T(s) = 5.41$</p>
 <p>(e)</p>	 <p>No. Of Regions=334, $F(I) = 143.17$, $F'(I) = 87.65$, $Q(I) = 233.74$, $T(s) = 5.29$</p>

Table 3 Average quantitative results for 50 standard images

Algorithm Parameter	A2DKM	A2DFCM-KM
F1	0.76	0.1043
F2	0.15	0.0585
Q1	0.76	0.1193
T(s)	0.67	0.0061

The experimental results prove that the A2DFCM-KM algorithm produces enhanced segmented images when compared with other clustering algorithms.

6. Conclusion:

In this paper, we have presented an automatic fuzzy C-means clustering algorithm depend on the spatial similarity information. A conventional fuzzy c-means (FCM) technique does not employ the spatial information in the image. The A2DFCM-KM algorithm that enhances spatial information into the membership function is used for clustering, although a conventional FCM method does not fully exploit the spatial information in the image. The experimental results show that our method can do better other segmentation methods for the images. We enhanced the quality of image during segmentation and also we reduced the time process. The proposed A2DFCM-KM algorithm could be applied to color images, where the robustness of the algorithm can be improved, in that way further behind its high effectiveness as an unsupervised segmentation technique.

REFERENCES

- [1] S. K. Fu and J.K. Mui, "A survey on image segmentation", Pattern Recognition, Vol 13, Issue 1, Pages 3–16, 1981
- [2] R. M. Haralick, L.G. Shapiro, "Image segmentation techniques", Computer Vision Graphics Image Process, Applications of Artificial Intelligence II, April 1985.
- [3] N. Pal and S. Pal, "A review on image segmentation technique", Pattern Recognition, Vol 26, Issue 9, pp 1277–1294, September 1993.
- [4] W. Skarbek, A. Koschan, "Colour image segmentationa survey", Tech. Rep, 32, (1994).
- [5] Y.J. Zhang, "A survey on evaluation methods for image segmentation", Pattern Recognition, 29, 1335.1346 (1996).
- [6] H.D. Cheng, X.H. Jiang, Y. Sun, and J. Wang, "Colour image segmentation: advances and prospects", Pattern Recognition, 34, 2259.2281(2001).
- [7] Weina Wang, Yunjie Zhang, Yi Li, Xiaona Zhang, "The global fuzzy c-means clustering algorithm", [In] Proceedings of the World Congress on Intelligent Control and Automation, Vol. 1, pp. 3604–3607, 2006.
- [8] L.A.Zadeh, "Fuzzy sets", Information and Control, Vol. 8, pp. 338–353, 1965.
- [9] J.C.Bezdek, "Pattern Recognition with Fuzzy Objective Function Algorithms", Plenum Press, NewYork, 1981.

- [10] J.C.Bezdek, L.O.Hall, L.P.Clarke, "Review of MR image segmentation techniques using patternRecognition", Medical Physics20(4), pp. 1033–1048, 1993.
- [11] N. Ferahta, A.Moussaoui, K.Benmahammed, V.Chen, "New fuzzy clustering algorithm appliedto RMN image segmentation", International Journal of Soft Computing 1, pp. 137–142, 2006.
- [12] M. Ahmed, S. Yamany, N. Mohamed, A. Farag, and T. Moriarty, "Amodified fuzzy C-means algorithm for bias field estimation and segmentation of MRI data", IEEE Trans. Med. Imag., vol. 21, pp. 193–199, 2002.
- [13] Sudhavani and Sathyaprasad, "Segmentation of Lip Images by Modified Fuzzy C-means Clustering Algorithm", IJCSNS International Journal of Computer Science and Network Security, Vol.9 No.4, pp.187-191, April 2009.
- [14] Kannan, Ramathilagam and Sathya, "Robust Fuzzy C-Means in Classifying Breast Tissue Regions", In proceedings of ARTCOM International Conference on Advances in Recent Technologies in Communication and Computing, pp.543-545,2009.
- [15] Huynh Van Lung and Jong-Myon Kim, "A generalized spatial fuzzy Cmeans algorithm for medical image segmentation", In proceedings of the 18th International conference on Fuzzy Systems, pp.409-414, 2009.
- [16] Ruoyu Du and Hyo Jong Lee, "A modified-FCM segmentation algorithm for brain MR images", In proceedings of ACM International Conference on Hybrid Information Technology, pp.25-27, 2009.
- [17] Huiyu Zhou, Gerald Schaefer, Abdul Sadka,EmreCelebi, "Anisotropic mean shift based fuzzy c-means segmentation of skin lesions", Proceedings of the 5th international conference on Soft computing as trans disciplinary science and technology, pp.438-443, 2008.
- [18] J.C. Bezdek, Pattern Recognition with Fuzzy Objective Function Algorithms, New York, Plenum Press, 1981.
- [19] J.C. Dunn, .A fuzzy relative of the ISODATA process and its use in detecting compact well separated clusters., J. Cybernetics 3, 32.57 (1974).
- [20] K. Chen, Y. Ma, L. Jun, "Segmentation by fusion of selfadaptive SFCM cluster in multi-color space components", International Journal of Image Processing (IJIP),6 ,pp157–166. 2012
- [21] K.S. Chuang, H.L. Tzeng, S. Chen, J. Wu, T.J. Chen, "Fuzzy C-means clustering with spatial information for image segmentation", Computerized Medical Imaging and Graphics,Vol 30, Issue 1, Pages 9–15, January 2006.
- [22] F. A. Jassim, "Hybrid Image Segmentation using Discerner Cluster in FCM and Histogram Thresholding", International Journal of Graphics & Image Processing Vol.2 Issue 4 November 2012.
- [23] KiriWagstaff, Claire Cardie , Seth Rogers , Stefan Schroedl, "Constrained K-means Clustering with Background Knowledge",ICML, 2001

- [24] P. S. Bradley , Usama M. Fayyad, “Refining Initial Points for K-Means Clustering”,ICML, 1998
- [25] Aristidis Likas, Nikos Vlassis, Jakob J. Verbeek, “The global k-means clustering algorithm”, Vol 36, Issue 2, Pages 451–461, February 2003
- [26] Yusoff, I. A., Isa, N. A. M., & Hasikin, K. (2013). Automated two-dimensional K-means clustering algorithm for unsupervised image segmentation. Computers & Electrical Engineering, 39(3), 907-917.

